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ScienceDirect
International Journal of Multiphase Flow 33 (2007) 675–689
International Journal of Multiphase Flow
www.elsevier.com/locate/ijmulflow

Shear-thinning fluids flow in fixed and fluidised beds

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Received 8 February 2006; received in revised form 14 December 2006

Abstract

In the paper the results of experimental studies directed on the effect of liquids' properties (aqueous solutions of polymers and surfactants) on resistance of the flow through porous and fluidised beds, are presented. It was shown that the determination of the values of minimal fluidisation velocity on the basis of an analysis of pressure drop related to the current two-phase system height gives the more accurate values than the method based on the initial bed height. Independently of the Newtonian or shear-thinning properties of the liquid flowing through motionless or fluidised bed, the relation of the friction factor on well-defined Reynolds number (related to real rheological parameters of a liquid studied) is analogous. It has been shown that the diagram proposed by Koziol et al. can be stated as the generalized one, not only for the determination of the solid particles motion in Newtonian fluids, but for the shear-thinning liquids too. In the last case it should be taken into account that the critical value of porosity cannot be taken equal to 0.4, but should be appropriate to the real porosity in the critical conditions for a given system solid particle–liquid. The generalization of both, the map of Bi and Grace related to the characteristic fluidisation ranges and the diagram of the classification of particles fluidised proposed by Goossens for gas-fluidisation, on any systems of solid particles–power law fluids, has been proposed.

Keywords: Solid–liquid systems; Power-law fluids; Polymers; Surfactants; Friction factor; Minimum fluidisation velocity

1. Introduction

Many processes are enhanced by the successful application of fluidisation, which compared to other processes, offers improved fluid–solids contact, near isothermal conditions, and improved heat and mass transfer. Fluidisation starts at a point when the bed pressure drop exactly balances the net downward forces on the bed packing, thus in that point the system of solid particle–Newtonian fluid obeys the force balance relations:

$$\frac{\Delta P}{H} = g \cdot (\rho_p - \rho_L) \cdot (1 - \varepsilon_{crit}), \tag{1}$$

$$\frac{150 \cdot w_{0,crit} \cdot \eta_L \cdot (1 - \varepsilon_{crit})}{\varepsilon_{crit}^3 \cdot d_p^2} + \frac{1.75 \cdot \rho_L \cdot w_{0,crit}^2}{\varepsilon_{crit}^3 \cdot d_p} = g \cdot (\rho_p - \rho_L), \tag{2}$$

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0301-9322/$ - see front matter
doi:10.1016/j.ijmultiphaseflow.2006.12.004

where ΔP is the pressure drop, H is bed height, g is gravity acceleration, ρ_p is particle density, ρ_L is fluid density, ε_{crit} is current porosity corresponding to so-called minimum fluidisation velocity, $w_{0,crit}$ is superficial velocity of a fluid defined as the volumetric flow divided by the cross sectional area corresponding to minimum fluidisation velocity, η_L is Newtonian fluid viscosity, d_p is particle diameter. Therefore the result is the relation

$$\frac{\Delta P}{H}=\frac{150\cdot w_{0,crit}\cdot\eta_0\cdot(1-\varepsilon_{crit})^2}{\varepsilon_{crit}^3\cdot d_p^2}+\frac{1.75\cdot(1-\varepsilon)\cdot w_{0,crit}^2\cdot\rho_0}{\varepsilon_{crit}^3\cdot d_p}. \tag{3}$$

The two terms on the right hand side of Eq. (3) can be recognized as viscous and inertial contributions. In most industrial applications involving fluidised beds, the solid particles diameters and also their total volume V_s in two-phase system are small. In these cases, the second term in Eq. (2) is negligible compared to the first one, so that

$$\frac{150\cdot w_{0,crit}\cdot\eta_L}{g\cdot(\rho_p-\rho_L)\cdot d_p^2}=\frac{\varepsilon_{crit}^3}{1-\varepsilon_{crit}}. \tag{4}$$

For a given bed the above equation can be used for both, the unexpanded (of porosity ε_0 and for superficial velocity w_0) and the expanded, states. As total volume of two-phase system increases ε may increase and hold the pressure drop ΔP constant (H will also increase but its effect is much less than the effect of change in porosity ε). At velocities w_0 less than the minimum fluidisation velocity $w_{0,crit}$ the bed behaves as a packed bed. However, as the velocity is increased above $w_{0,crit}$, not only the bed does expand (H increases), but also the particles move apart, and ε also increases to keep the ΔP constant. The equations derived for minimum fluidisation velocity can be applied to liquids as well as gases, but beyond the minimum fluidisation velocity $w_{0,crit}$, the appearance of beds with liquids or gases is quite different.

The fundamental tasks of fluidised-bed dynamics are to determine the condition of transition from a fixed-bed into a fluidised-bed state and the prediction of bed expansion in relation to rheological properties of liquid media. In some publications (Richardson, 1971; Dullien, 1975; Rietema, 1982; Joshi, 1983; Kunii and Levenspiel, 1990; di Felice, 1995; Jamialahmadi and Muller-Steinhagen, 2000) the previous findings have been reviewed where Newtonian liquid fluidisation is a specific case, such as the problem of flow in porous media, dispersed two-phase systems as well as particulate fluidisation. In papers of Kemblowski et al. (1989) and Chhabra et al. (2001) most experimental and theoretical studies concerned with pressure drop determination for non-Newtonian fluid flow through porous media have been reviewed. In a paper by Chhabra and Srinivas (1991) an attempt has been made to reconcile and critically analyse the voluminous literature available on the flow of rheologically complex fluids through unconsolidated fixed bed and fluidised bed. Chhabra (1993) has demonstrated that non-Newtonian liquid fluidised beds in literature have received limited attention. The representative summary on the variety of packings and non-Newtonian fluids used as well as the representative studies on fluidisation of particles with non-Newtonian fluids was presented in paper of Chhabra et al. (2001). The bulk of the information available in the literature relates to the beds of spherical particles fluidised by power-law inelastic polymer solutions. Little is known about the role of fluid viscoelasticity – due to mobility of particles in a fluidised bed, the viscoelasticity manifests itself in different ways in fixed and fluidised beds. While in the creeping flow through fixed bed of particles the viscoelasticity gives rise to excess pressure drop, in the flow through fluidised beds it can lead to segregation of particles. The last review paper of Chhabra et al. (2001) suggests that the knowledge is very scant about the detailed kinematics of flow including flow patterns, residence time distribution, micro-level phenomena such as polymer adsorption, retention and wall effects. The current literature has not failed to give special attention to the solutions of surface-active agents flowing through fixed and fluidised bed.

The present study is concerned with the experimental comparison of the liquid phase properties effect on both, friction factors for the flow of the rheostable fluids through porous and fluidised bed, and on minimum fluidisation velocities for Newtonian and non-Newtonian (both surfactants and polymers) aqueous solutions.

2. Experimental

Fig. 1 illustrates the schematic diagram of the experimental set-up. The main element of the test installation was an apparatus constructed from organic glass pipe with an inside diameter of $T = 0.090$ m and a height of

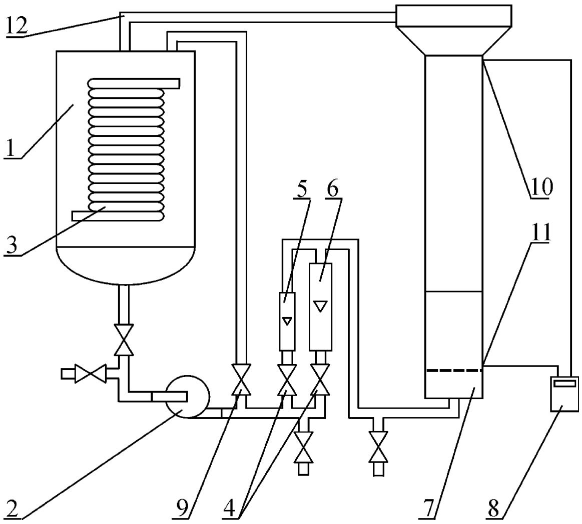

Fig. 1. Experimental set-up: 1 – tank; 2 – pump UPE 25-60B; 3 – heat exchanger; 4, 9 – control valves; 5, 6 – rotameters; 7 – fluidised column; 8 – numerical differential manometer Manoport 3922; 10, 11 – pressure probe.

Table 1
Characteristics of the agalite particles studied

Fraction no.	Range of d_p (mm)	Averaged solid diameter d_p (m)	Primary porosity of a bed ε_0 (m^3/m^3)	Density ρ_s (kg/m^3)	Reduced diameter in a water $d_{p,r} = \frac{d_p}{\delta_{e,w}}$
1	1.5–2	0.0018	0.421	2110	38.49
2	2–2.5	0.0023	0.418	2236	49.18
3	2.5–3	0.0028	0.415	2364	59.88
4	3–4	0.0036	0.416	2282	76.99
5	4–5	0.0044	0.422	2258	94.10

2.0 m equipped in the scale for determination of fluidised bed height. The fluidising column was equipped with probes connected to a piezoelectric pressure transducer. The numerical differential manometer used permitted to obtain the experimental data with accuracy $\pm 0.2\%$ of full scale. The idea of the measurements is analogous to this applied by di Felice (2002). In literature the various techniques for determination of minimum fluidisation velocity (e.g., light transmission technique: Didwania and Homsy, 1981; conductivity method: Briens et al., 1997; radioactive method: Chen et al., 2001) have been used.

The studied beds were composed of agalite balls of characteristics shown in Table 1. The liquids tested were circulated through the fixed beds using a pump with maximal volume flow rate 4000 (l/h). The power-law characteristics

$$\tau = K \cdot \dot{\gamma}^n, \tag{5}$$

where τ is the shear stress, K is consistency coefficient, n is flow behaviour index, of the liquid systems studied are presented in Table 2. The process temperature of 293 K in all measurements was regulated by a heat exchanger and controlled at the end of the test section. The rotameters have been scaled for each of non-Newtonian liquids. Because the flow through a rotameter can involve the significant extension component and hence the polymer solutions can suffer flow induced degradation, the rheological properties of the non-Newtonian liquids used were controlled before and past rotameter calibration as well as before and past fluidisation. In polymer aqueous solutions used the mechanical degradation has not been observed. The direct measured quantities were: the pressure drop ΔP, volume liquid rate V, dynamic height of a bed H and process temperature T. The ranges of the change of the characteristic process and geometrical quantities tested are

Table 2
Characteristics of power-law solutions studied

Liquid	Concentration (ppm)	Parameters of the power-law $\tau = K \cdot \dot{\gamma}^n$		Density ρ_L (kg/m^3)	Equivalent liner dimension $\delta_{e,n}10^5$ (m)	Equivalent liquid velocity $\omega_{e,n}$ 10^2 (m/s)
		K, η (Pa s^n)	n			
Water	–					
Aqueous solutions of	400	0.001	1	998.2	4.676	2.142
TWEEN 60	800					
	1200	0.001064	0.99	1000	4.679	
Aqueous solutions of	100	0.001	1	998.2	4.676	
PAA of molar	200					
weight of $M_p \approx 10^3$	500	0.00106		1000	4.853	2.183
(PAA)	3000	0.0012		1001	5.272	2.274
Aqueous solutions of	30	0.00106	1	998.2	4.862	2.184
PAA of molar	100	0.00136	0.98	1000	5.289	2.278
weight of	200	0.00183	0.96		5.955	2.417
$M_p \approx 2.10^6$ with	500	0.00385	0.9	1002	7.746	2.757
special additives	1500	0.013	0.82	1001	13.17	3.594
(Rokrysol WF1)						
Aqueous solutions of	3000	0.053	0.75	1002	27.48	5.192
Na–CMC	6000	0.221	0.67	1002	58.49	7.575
	9000	0.55	0.65	1003	108.07	10.30

PAA – polyacrylamide, NA-CMC – carboxymethylcellulose sodium salt, TWEEN 60 – polyoxyethylene (20) sorbitan monostearate.

Table 3
The ranges of the change of characteristic process and geometrical quantities tested

Quantity	Dimension	Newtonian liquids	Power-law fluids
Equivalent linear dimension for liquid	m	$\delta_{e,n} \in (0.00004676, 0.00005272)$	$\delta_{e,n} \in (0.00005289, 0.001081)$
Equivalent velocity of a liquid	m/s	$\omega_{e,n} \in (0.02142, 0.02184)$	$\omega_{e,n} \in (0.02278, 0.1030)$
Superficial liquid velocity	m/s	$w_0 \in (0.004369, 0.2295)$	$w_0 \in (0.00318, 0.227)$
Reduced velocity of a liquid	–	$w_{0,r} \in (0.2001, 10.51)$	$w_{0,r} \in (0.049, 9.993)$
Reduced particle diameter	–	$d_{p,r} \in (34.15, 94.08)$	$d_{p,r} \in (1.666, 83.20)$
Pressure drop	Pa	$\Delta P \in (15, 2300)$	$\Delta P \in (25, 2080)$
Unitary pressure drop	Pa/m	$\frac{\Delta P}{H} \in (180, 7300)$	$\frac{\Delta P}{H} \in (250, 7353)$
Euler number	–	$Eu \in (17.74, 10644)$	$Eu \in (16.98, 75860)$
Reynolds number	–	$Re_{0,n} \in (10, 8960)$	$Re_{0,n} \in (0.4530, 5230)$

presented in Table 3. The exemplary experimental data are presented in Fig. 2. The relationships of $\Delta P = f(w_0, H)$ and $\Delta P/H = f(w_0)$ obtained were similar for all solid–liquid systems tested. It has been shown that the critical velocity of fluidisation $w_{0,\text{crit}}$ is independent of the height H (Chhabra et al., 2001). The

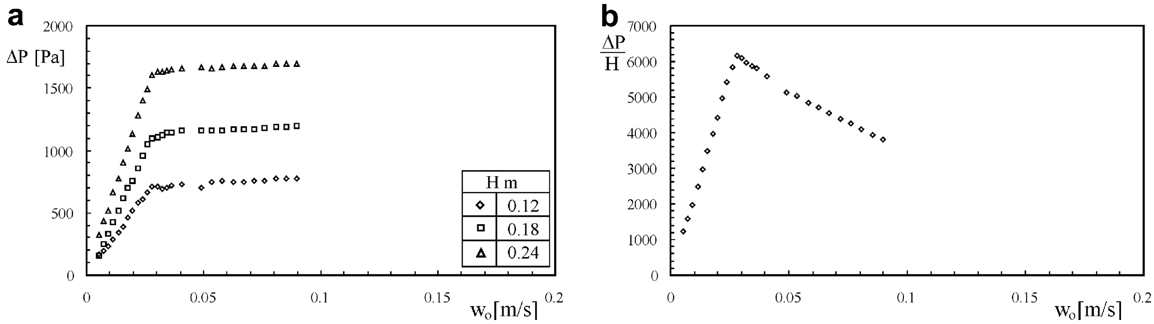

Fig. 2. The experimental results on pressure drop for the Rokrysol WF1 aqueous solution of concentration of 1500 ppm for the bed of average diameter $d_p = 4.4 \cdot 10^{-3}$ m.

determination of the values of minimal fluidisation velocity on the basis of an analysis of pressure drop related to the current two-phase system height gives the more accurate values than this one on the basis of the initial bed height. For all experimental data the friction factor values λ were calculated on the basis of relationships as follows:

$$\lambda = 2 \cdot \frac{\Delta P}{w_0^2 \cdot \rho_L} \cdot \frac{d_p}{H} \cdot \frac{\varepsilon^3}{1-\varepsilon}, \tag{6}$$

where ε is the current bed porosity when the initial one is equal to ε_0 (static bed voidage), defined as

$$\varepsilon = 1 - \frac{H}{H_0} \cdot (1 - \varepsilon_0). \tag{7}$$

For power-law fluids studied the characteristic modified Reynolds number definition used was taken as proposed by Kemblowski et al. (1989):

$$Re_{0,n} = \frac{w_0^{2-n} \cdot d_p^n \cdot \rho_L}{K \cdot (1-\varepsilon)^n} \cdot \left(\frac{4n}{1+3n}\right)^n \cdot \left(\frac{15 \cdot \sqrt{2}}{\varepsilon^2}\right)^{1-n}, \tag{8}$$

which for the Newtonian liquid ($K = \eta_L$, $n = 1$) is reduced to the classical one

$$Re_{0,N} = \frac{w_0 \cdot d_p \cdot \rho_L}{\eta_L \cdot (1-\varepsilon)}. \tag{9}$$

The relationship of the $\lambda = f(Re_{0,n})$ obtained for all the complex liquid–solid particles systems studied as well as for the liquid flow through packed and fluidised bed, is presented in Fig. 3. The solid line represents the generalized Ergun's as well as Blake–Kozeny formulas proposed for the Newtonian liquid flow through packed and fluidised bed. It has been found that the values of drag coefficient in both, the laminar and transitional ($Re_{0,n} < 165$), ranges of the flow for all fluids studied are determined by the generalised Ergun equation

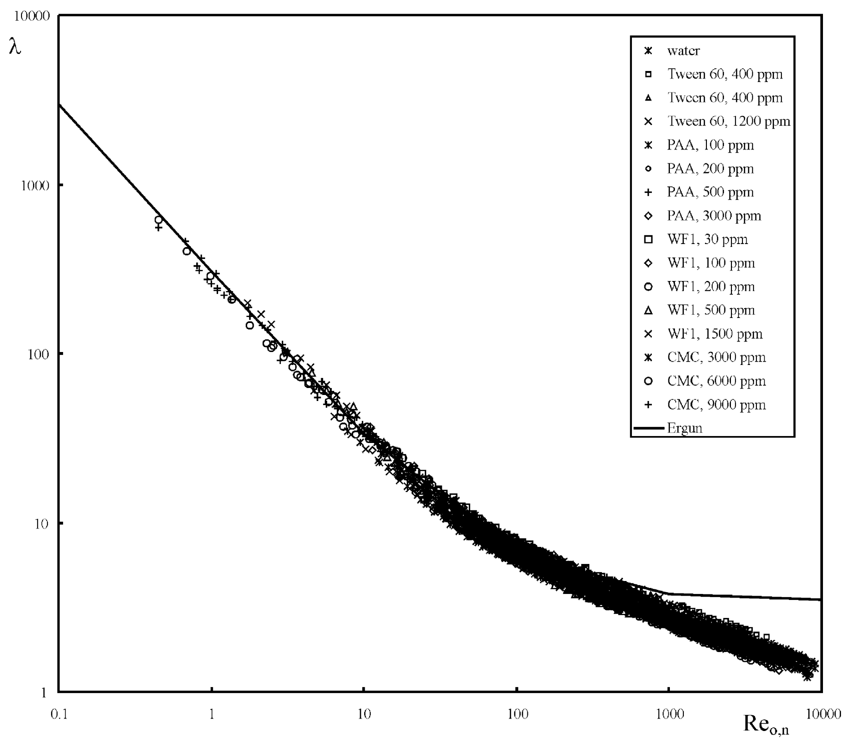

Fig. 3. Friction factor versus modified Reynolds number (8).

$$\frac{\Delta P}{H} = \left(\frac{300}{Re_{0,n}} + 3.5\right) \cdot \frac{w_0^2 \cdot \rho_L}{2} \cdot \frac{1-\varepsilon}{\varepsilon^3 \cdot \varphi_p} \cdot \frac{1}{d_p}, \tag{10}$$

where φ_p is the particle shape factor (in the study presented $\varphi_p = 1$), and

$$\lambda = \frac{300}{Re_{0,n}} + 3.5. \tag{11}$$

The expression (11) confirms the previous suggestions for the non-Newtonian purely viscous fluid flow through packed bed of Chhabra and Srinivas (1991) and Srinivas and Chhabra (1992) who proposed an adaptation of Ergun equation for spherical and non-spherical particles. It was found that the Eq. (11) gives the satisfactory correlation of their data with weak inertial effects. The experimental data presented show the possibility to extend the relation mentioned above to fluidised bed too. For the values of $Re_{0,n} > 165$ the friction factor values

$$\lambda = 29.5 \cdot Re_{0,n}^{-0.34}, \tag{12}$$

are lower than resulting from the both, Ergun and Burke–Plummer, relationships:

$$\frac{\lambda}{\lambda_{\text{Ergun}}} = 1.8247 - 0.1589 \cdot \ln Re_{0,n}. \tag{13}$$

The comparison of the friction factor values (12) with the literature data was not possible. We did not find the experimental data for power-law fluids flow through packed and fluidised bed related to the range of Reynolds number greater than 165.

Comiti et al. (2000) extended their previous work (Sabiri and Comiti, 1997) for Newtonian fluids flow through porous media to inelastic non-Newtonian fluids. Taking into account that the bed can be considered as a bundle of identical tortuous pores of diameter d_{pore} and length L_{pore} and H is the height of two-phase system, the tortuosity of the porous medium is defined as

$$\tau_{\text{pore}} = \frac{L_{\text{pore}}}{H}. \tag{14}$$

The diameter of the pores is calculated by setting that their developed surface area is identical to the fixed bed surface actually reached by the fluid flow:

$$d_{\text{pore}} = \frac{4\varepsilon}{a_{\text{vd}} \cdot (1-\varepsilon)}, \tag{15}$$

where a_{vd} is the dynamic specific surface area of the porous medium for spherical particles determined from relationship:

$$a_{\text{vd}} = \frac{6}{d_p}. \tag{16}$$

The average pore velocity is related to the superficial velocity as

$$w_{\text{pore}} = \frac{w_0}{\varepsilon} \cdot \tau_{\text{pore}}, \tag{17}$$

where tortuosity for tightly packed spheres is approximated (Comiti and Renaud, 1989) by

$$\tau_{\text{pore}} = 1 - 0.41 \cdot \ln \varepsilon. \tag{18}$$

The general equation of a model proposed by Sabiri and Comiti (1997) can be written using the following dimensionless form (Chhabra et al., 2001):

$$f_{\text{pore}} = \frac{16\alpha}{Re_{\text{pore}}} + 0.194\beta, \tag{19}$$

where the Reynolds number of pore based on conduit flow

$$Re_{\text{pore}} = \frac{(w_0 \cdot \tau_{\text{pore}})^{2-n} \cdot \rho_{\text{L}} \cdot \varepsilon^{2(1-n)}}{2^{n-3} \cdot K \cdot \left(\frac{1+3n}{4n}\right)^n \cdot (1-\varepsilon)^n \cdot a_{\text{vd}}^n} \tag{20}$$

and friction factor is defined as

$$f_{\text{pore}} = \frac{\Delta P}{H} \cdot \frac{2\varepsilon^3}{w_0^2 \cdot \rho_{\text{L}} \cdot \tau_{\text{pore}}^3 \cdot (1-\varepsilon) \cdot a_{\text{vd}}}. \tag{21}$$

Taking into account relationship (16) for spherical particles the formulas (20) and (21) take the forms as follows:

$$Re_{\text{pore}} = \frac{w_0^{2-n} \cdot d_{\text{p}}^n \cdot \rho_{\text{L}}}{K \cdot \left(\frac{1+3n}{4n}\right)^n} \cdot \frac{\varepsilon^{2(1-n)}}{(1-\varepsilon)^n} \cdot \frac{\tau_{\text{pore}}^{2-n}}{2^{2n-3} \cdot 3^n} \tag{22}$$

and

$$f_{\text{pore}} = \frac{\Delta P}{H} \cdot \frac{d_{\text{p}}}{6} \cdot \frac{2}{w_0^2 \cdot \rho_{\text{L}}} \cdot \frac{\varepsilon^3}{\tau_{\text{pore}}^3 \cdot (1-\varepsilon)}, \tag{23}$$

α and β in relationship (19) are two coefficients for wall effects defined by

$$\alpha = \left[1 + \frac{4}{a_{\text{vd}} \cdot T \cdot (1-\varepsilon)}\right]^{1+n} = \left[1 + \frac{2d_{\text{p}}}{3 \cdot T \cdot (1-\varepsilon)}\right]^{1+n}, \tag{24}$$

$$\beta = \left(1 - \frac{d_{\text{p}}}{T}\right)^2 + 0.427 \cdot \left[1 - \left(1 - \frac{d_{\text{p}}}{T}\right)^2\right]. \tag{25}$$

It has been shown (Chhabra et al., 2001) that for flow behaviour index values changed from 0.27 to 0.91, the porosity in the range $0.31 \leqslant \varepsilon \leqslant 0.46$ and the ratio of $d_{\text{p}}/T < 0.15$ (in the study presented d_{p}/T changed from 0.02 to 0.0489) the general Eq. (19) reduces to

$$f_{\text{pore}} = \frac{16}{Re_{\text{pore}}} + 0.194. \tag{26}$$

Comparison of the formulas of Reynolds number (8) and (22) as well as the definitions of friction factor λ (6) and f_{pore} (23) gives the relationships

$$Re_{\text{pore}} = \frac{\tau_{\text{pore}}^{2-n}}{3 \cdot 2^{\frac{3n-5}{2}} \cdot 5^{1-n}} \cdot Re_{0,n}, \tag{27}$$

$$f_{\text{pore}} = \frac{\lambda}{6 \cdot \tau_{\text{pore}}^3}. \tag{28}$$

Substituting (27) and (28) into expression (26) permits to obtain the equation as follows:

$$\lambda = \frac{254.6 \cdot (\sqrt{2})^{3n}}{5^n \cdot Re_{0,n}} \cdot \tau_{\text{pore}}^{1+n} + 1.164 \cdot \tau_{\text{pore}}^3. \tag{29}$$

The procedure based on the capillary model (Sabiri and Comiti, 1997) needs the knowledge of tortuosity. The review of existing in the literature proposals regarding the value and the meaning of the tortuosity factor τ_{pore} was given in paper of Chhabra et al. (2001). It results from there that various authors suggested various correlations of $\tau_{\text{pore}} = f(\varepsilon)$ and the divergences between values of τ_{pore} (Fig. 4) can be great. For example for $\varepsilon = 0.44$ by Mauret and Renaud (1997) the value of τ_{pore} is equal to 1.402 whereas from a paper of Foscolo et al. (1983) $\tau_{\text{pore}} = 2.273$, respectively. Taking into account the relation (18) and a fact that in the experiments presented the porosity changed from 0.415 (for packed bed) to 0.44 (for fluidised bed), it can be reasonable for the needs of comparative analysis τ_{pore} to be changed from 1.337 to 1.361. For $\tau_{\text{pore}} = 1.337$ the formula (29) takes the form

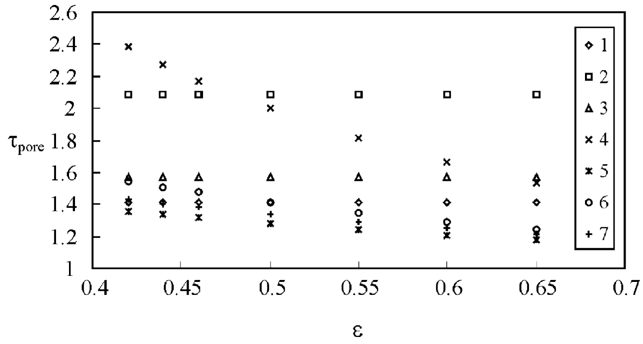

Fig. 4. Comparison of the literature results on tortuosity of fixed and fluidised bed: 1 – Carman (1956); 2 – Christopher and Middleman (1965); 3 – Sheffield and Metzner (1976); 4 – Foscolo et al. (1983); 5 – Comiti and Renaud (1989); 6 – Puncochar and Drahos (1993) and 7 – Mauret and Renaud (1997).

$$\lambda = \frac{340.4 \cdot 0,756^n}{Re_{0,n}} + 2.782 \tag{30}$$

and for $\tau_{pore} = 1.361$

$$\lambda = \frac{346.5 \cdot 0,770^n}{Re_{0,n}} + 2.934, \tag{31}$$

respectively. The exemplary comparison of the values of friction factor λ resulted from the expressions (11), (30) and (31) for flow behaviour index values of $n = 1$ and $n = 0.65$ is presented in Fig. 5. It was found that the values resulting from the capillary model of Sabiri and Comiti (1997) and from an adaptation of Ergun equation (Chhabra and Srinivas, 1991; Srinivas and Chhabra, 1992) are comparable. Taking into account that the last proposal includes the quantities directly measured, in our opinion, it is more usable in design practice.

The experimental data on minimum fluidisation velocity are presented in Fig. 6. It has been shown that the minimum fluidisation velocity depends on particle diameter as well as the rheological characteristics of the liquid phase. The introduction of the reduced particle diameter $d_{p,r}$, defined as

$$d_{p,r} = \frac{d_p}{\delta_{e,n}}, \tag{32}$$

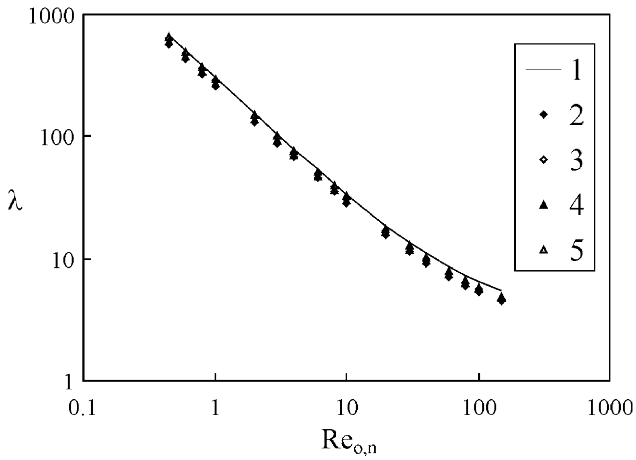

Fig. 5. Comparison of the authors' experimental data on friction factor values versus modified Reynolds number with resulting from capillary model: 1 – present data, Eq. (11); 2 – capillary model, Eq. (30), $n = 1$; 3 – capillary model, Eq. (30), $n = 0.65$; 4 – capillary model, Eq. (31), $n = 1$; 5 – capillary model, Eq. (31), $n = 0.65$.

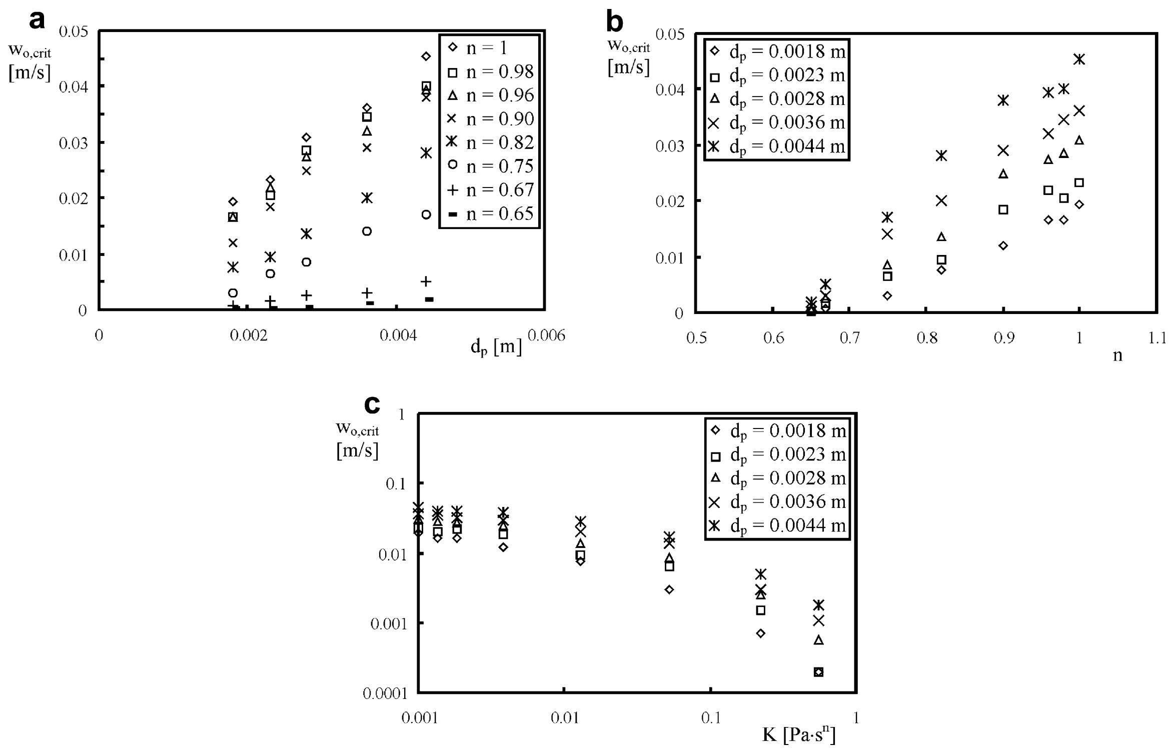

Fig. 6. Effect of particle diameter, flow behaviour index and consistency coefficient on the minimum fluidisation velocity $w_{0,\mathrm{crit}}$ for various power-law liquids studied.

where modified equivalent linear dimension $\delta_{\mathrm{e,n}}$ is described by the relation

$$\delta_{\mathrm{e,n}} = \left(\frac{K^2}{g^{2-n}\cdot\rho_{\mathrm{L}}^2}\right)^{\frac{1}{2+n}} \tag{33}$$

permitted to analyse the experimental data (Fig. 7) on the basis of the dimensionless relationships. For that purpose the reduced critical value of fluidisation velocity

$$w_{0,\mathrm{crit,r}} = \frac{w_{0,\mathrm{crit}}}{\omega_{\mathrm{e,n}}}, \tag{34}$$

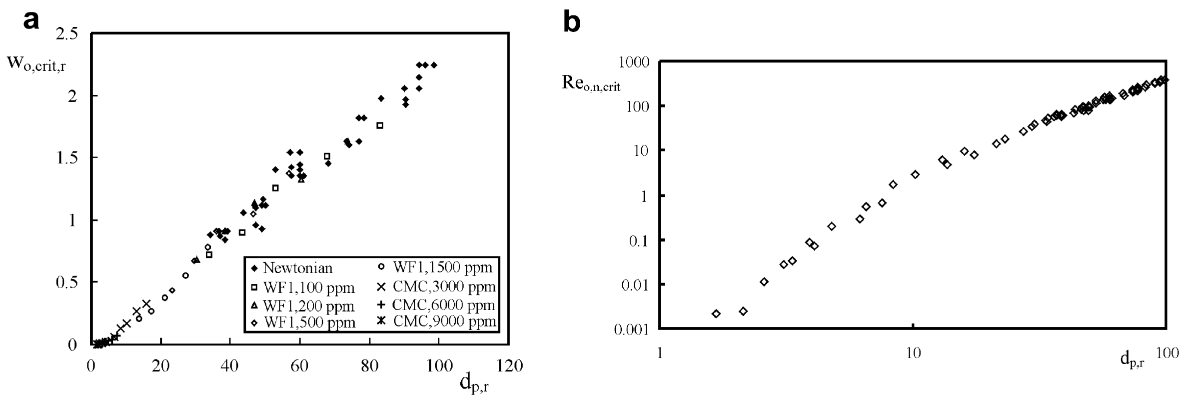

Fig. 7. The relations of $w_{0,\mathrm{crit,r}} = f(d_{\mathrm{p,r}})$ and $Re_{0,\mathrm{n,crit}} = f(d_{\mathrm{p,r}})$.

where the equivalent fluid velocity for power-law liquids is defined as

$$\omega_{\mathrm{e,n}} = \left(\frac{g^n \cdot K}{\rho_{\mathrm{L}}}\right)^{\frac{1}{2+n}} \tag{35}$$

and Reynolds number

$$Re_{0,\mathrm{n,crit}} = \frac{w_{0,crit}^{2-\mathrm{n}} \cdot d_{\mathrm{p}}^n \cdot \rho_{\mathrm{L}}}{K \cdot (1-\varepsilon_{\mathrm{crit}})^n} \cdot \left(\frac{4n}{1+3n}\right)^n \cdot \left(\frac{15 \cdot \sqrt{2}}{\varepsilon_{\mathrm{crit}}^2}\right)^{1-n}, \tag{36}$$

as the function of the reduced particle diameter (32), have been determined. The equivalent quantities (32) and (35) for Newtonian fluids reduce to well-known ones, proposed by Hobler (1966) and Ramm (1976):

$$\delta_{\mathrm{e,n}} = \sqrt[3]{\frac{\eta_{\mathrm{L}}^2}{g \cdot \rho_{\mathrm{L}}^2}} \tag{37}$$

and

$$\omega_{\mathrm{e,n}} = \sqrt[3]{\frac{g \cdot \eta_{\mathrm{L}}}{\rho_{\mathrm{L}}}}. \tag{38}$$

The following correlation equations have been obtained:

- for $d_{\mathrm{p,r}} \in (3.08, 11.5)$

$$w_{0,\mathrm{crit}} = 0.000746 \cdot d_{\mathrm{p,r}}^{2.35} \pm 21.7\%, \tag{39}$$

- for $d_{\mathrm{p,r}} \in (11.5, 94.1)$

$$w_{0,\mathrm{crit}} = 0.0171 \cdot d_{\mathrm{p,r}}^{1.07} \pm 15.5\%. \tag{40}$$

In the next stage of analysis the results of present study have been compared with those of representative studies in the literature. Several authors compared their experimental data related to minimum fluidisation velocity with these ones for velocity of sedimentation observed. Taking it into account, first, the possibility of the use of the diagram proposed by Koziol et al. (1978) has been checked. Koziol et al. (1978) extended the work of Koziol (1974) in which the generalized dimensionless equation

$$\lambda_{\mathrm{s}} \cdot w_{\mathrm{s,r}}^2 = \frac{4}{3} \cdot \Gamma_\rho \cdot d_{\mathrm{p,r}}, \tag{41}$$

for terminal settling of single spherical particles in Newtonian fluids by means of reduced velocity

$$w_{\mathrm{s,r}} = \frac{w_{\mathrm{s}}}{\omega_{\mathrm{e,n}}} \tag{42}$$

and reduced particle diameter

$$d_{\mathrm{p,r}} = \frac{d_{\mathrm{p}}}{\delta_{\mathrm{e,n}}} \tag{43}$$

has been proposed. The Reynolds number was the product of the parameters:

$$Re_{\mathrm{s}} = \frac{w_{\mathrm{s}} \cdot d_{\mathrm{p}} \cdot \rho_{\mathrm{F}}}{\eta_{\mathrm{F}}} = w_{\mathrm{s,r}} \cdot d_{\mathrm{p,r}}, \tag{44}$$

where index F refers to any Newtonian fluid. On the basis of published experimental data the general modified plot $w_{\mathrm{s,r}} \cdot \Gamma_\rho^{-1/3} = f(d_{\mathrm{p,r}} \cdot \Gamma_\rho^{1/3})$ has been worked out (Koziol, 1974). This one was the generalization of the diagrams proposed by Schiller and Naumann (1933). In the paper of Koziol et al. (1978) as the base of proposal of generalized plot and equation for motion of spherical particles in Newtonian fluids there were taken the observations of Richardson and Zaki (1954) related to the ratio of sedimentation velocity w_ε to the settling velocity of single particle in a fluid w_s which can be written as

$$\frac{w_\varepsilon}{w_s} = \varepsilon^Z, \tag{45}$$

where exponent Z was found to be a function of Reynolds number and the ratio of particle diameter to column diameter d_p/T, and the analysis of the literature data on sedimentation of Rowe and Henwood (1961) and Wallis (1969). Rowe and Henwood (1961) proposed the following relationship between friction factor of spherical particles sedimentation λ_ε and friction factor of single particle settling λ_s:

$$\lambda_\varepsilon = \frac{\lambda_s}{\varepsilon^A} \tag{46}$$

and Wallis (1969) observed that

$$\lambda_\varepsilon \cdot Re_\varepsilon^2 = \lambda_s \cdot Re_s^2, \tag{47}$$

where the Reynolds number for sedimentation is defined as

$$Re_\varepsilon = \frac{w_\varepsilon \cdot d_p \cdot \rho_F}{\eta_F} = w_{\varepsilon,r} \cdot d_{p,r}. \tag{48}$$

Substituting the Reynolds numbers formulas (44) and (48) into (47) the equation:

$$\lambda_\varepsilon \cdot w_{\varepsilon,r}^2 = \lambda_s \cdot w_{s,r}^2 \tag{49}$$

has been obtained (Koziol et al., 1978). On the basis of published data in paper of Koziol et al. (1978) the generalized relationship

$$\lambda \cdot w_{r,\varepsilon}^2 = \frac{4}{3} \cdot \Gamma_\rho \cdot d_{p,r} \cdot \varepsilon^{4.65} \tag{50}$$

for spherical particles motion in any Newtonian fluid and dimensionless plot (Fig. 8) for prediction of process parameters, have been proposed. Koziol et al. (1978) suggested several lines to characterize the following processes: settling of the single particle in a fluid ($\varepsilon = 1$), sedimentation ($0.4 < \varepsilon < 1$) and fluidisation ($\varepsilon = \varepsilon_{crit} = 0.4$). In Fig. 8 the comparison of the present experimental data for fluidisation with the proposition of Koziol et al. (1978) has been presented. The characteristic values located in the diagram were taken as the reduced ones (32) and (34) for all power-law liquids (including Newtonian ones) used. From the pattern shown in Fig. 8 it results that the data from the present study are greater than resulting from the proposition of Koziol et al. (1978) for minimum fluidisation velocity. The effect is considered as a logical one. In studies presented the critical value of bed voidage was greater than 0.4 for all two-phase systems tested. The averaged

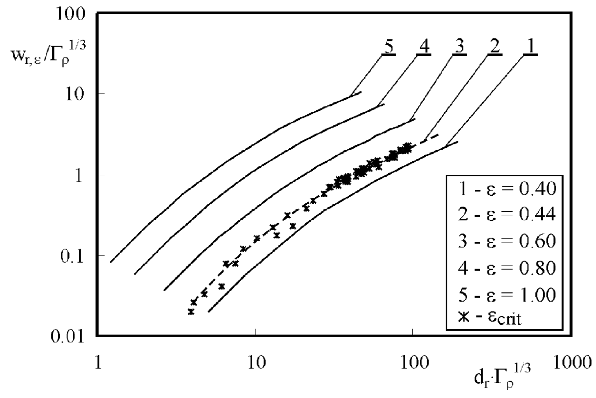

Fig. 8. The comparison of authors' results with the generalized diagram of the velocity of solid particle motion in Newtonian fluids proposed by Koziol et al. (1978).

value was $\varepsilon_{crit} = 0.442$. Therefore, it has been shown that the diagram proposed by Koziol et al. (1978) can be stated as the generalized one, not only for the determination of the solid particles motion in Newtonian fluids, but for the shear-thinning liquids too. In the last case it should be taken into account that the critical value of porosity cannot be taken equal to 0.4, but should be appropriate to the real porosity in the critical conditions for a given system solid particle–liquid.

Next, taking into account that the solid–gas fluidisation ranges map proposed by Bi and Grace (1995) includes two characteristic lines included in diagram of Koziol et al. (1978), the "place" of the authors' data in this diagram, has been analyzed too. From the pattern shown in Fig. 9 the results are as follows:

(1) The Bi and Grace (1995) proposal is useful not only for determination the characteristics in gas–solid fluidisation, but in power-law liquid–solid fluidisation too.
(2) The studies presented included the ranges of uniform fluidisation at $d_{p,r} \cdot \Gamma_\rho^{1/3} < 41.35$ and of the bubbling breached one at $d_{p,r} \cdot \Gamma_\rho^{1/3} > 41.35$.

The following step of the analysis was to determine the generalized method of the classification of fluidised particles in power-law fluids by generalized Archimedes number

$$Ar = d_{p,r}^3 \cdot \Gamma_\rho = d_{p,r}^3 \cdot \frac{\rho_s - \rho_L}{\rho_L} \tag{51}$$

similar to the proposed by Goossens (1998) for arbitrary Newtonian fluid–solid combination. The fundamental pattern of the Goosens proposal is presented in Fig. 10. In the C range limited from above by the Archimedes number value $Ar_1 = 0.97$ in total range of fluidisation there are predominant the phenomena connected with laminar flow (even when the local wires exist). For the solid particles which belong to class A the turbulence effect on the process kinetics is negligible one. The observation cannot be given for solid particles satisfying the conditions of the B range. The limit of the ranges A and B is described by the critical value of

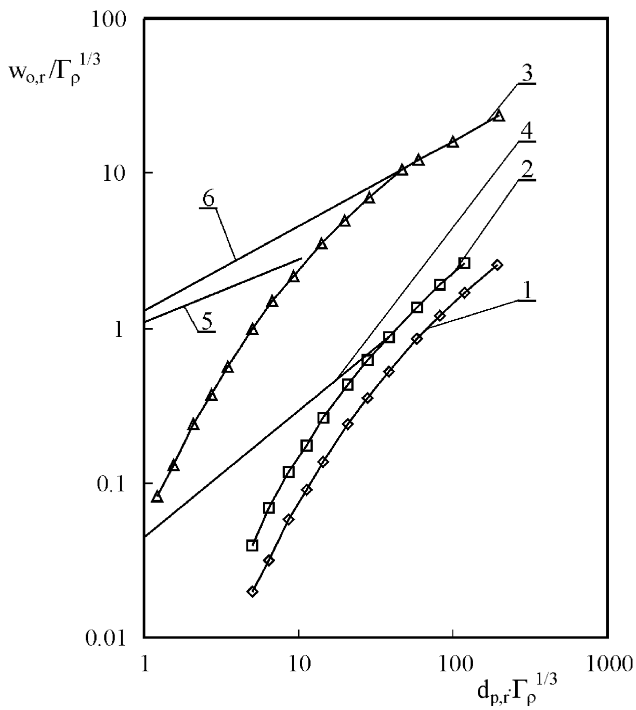

Fig. 9. Analysis of the possibility of generalization of the map of the fluidisation ranges (Bi and Grace, 1995) for $\varepsilon_{crit} = 0,442$: 1 – Koziol et al. (1978), fluidisation line for $\varepsilon = 0.4$; 2 – own results, ε_{crit}; 3 – Koziol et al. (1978), single particle settling; 4 – beginning of the bubble fluidisation (Bi and Grace, 1995), 5 – beginning of the turbulent fluidisation (Bi and Grace, 1995), 6 – beginning of hydraulic transport.

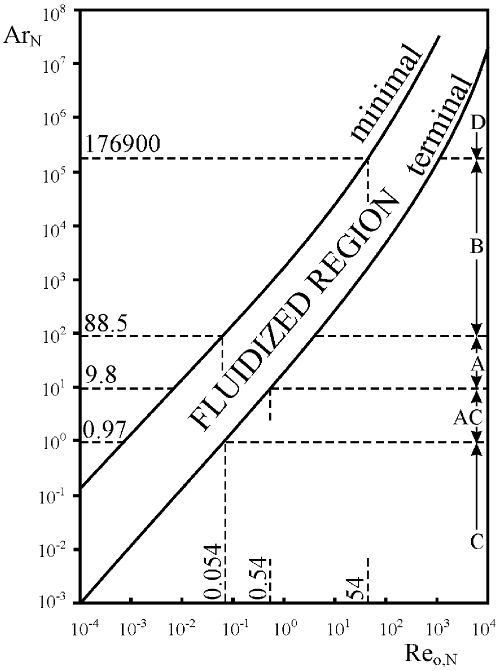

Fig. 10. General classification diagram for particles fluidised in Newtonian fluids (Goossens, 1998).

Archimedes number $Ar_2 = 88.5$. When the solid particles belong to the class D (from below limited by the value $Ar_3 = 176,900$) in total range of fluidisation the turbulent transfer effects play the dominating role. In the present study the experimental data are correlated by the following relationships:

- at $Re_{0,n,\mathrm{crit}} \in (0.011, 1.13)$

$$Ar_n = 488 \cdot Re_{0,n,\mathrm{crit}}^{0.71}, \tag{52}$$

- at $Re_{0,n,\mathrm{crit}} \in (1.13, 51.7)$

$$Ar_n = 460 \cdot Re_{0,n,\mathrm{crit}}^{1.20}, \tag{53}$$

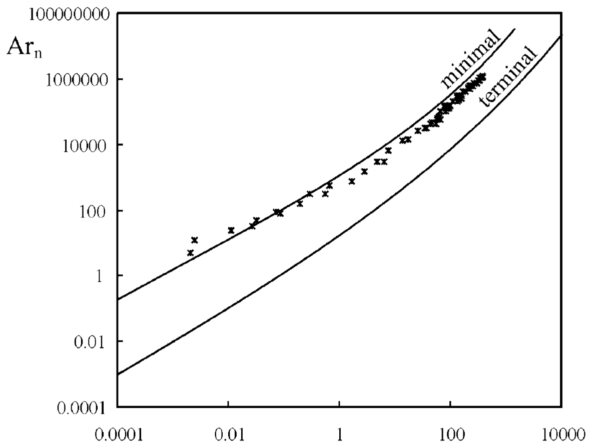

Fig. 11. The relation of Ar_n versus $Re_{0,n}$ from authors' experimental data for minimum fluidisation velocity compared with generalized limited lines in classification diagram (Goossens, 1998).

- at $Re_{0,n,\text{crit}} \in (51.7, 394)$

$$Ar_n = 120.3 \cdot Re_{0,n,\text{crit}}^{1.54}. \quad (54)$$

The patterns shown in Fig. 11 demonstrate that the experimental data obtained by us for minimum fluidisation velocity (except for the $Re_{0,n,\text{crit}} \leqslant 0.01135$, which can result from the accuracy of measurements for low values of pressure drop and fluidisation velocity) lie over the fluidised region proposed by Goossens (1998) when the axes are defined for Archimedes and Reynolds numbers characteristic for power-law fluid–solid particles systems studied. The suggestion can be made that the classification diagram of Goossens (1998) can be accepted as the general one for the arbitrary Stokes fluid–solid combination.

3. Conclusion

This study was concerned with the experimental comparison of the effect of liquid phase properties on minimum fluidisation velocities for shear thinning fluids and polymeric surface-active agent's solutions. It was shown that the determination of the values of minimal fluidisation velocity on the basis of an analysis of pressure drop related to the current two-phase system height gives more accurate values than this one based on the initial bed height. Independently of the Newtonian or shear-thinning properties of the liquid flowing through motionless or fluidised bed, the relation of the friction factor on well-defined Reynolds number (related to real rheological parameters of a liquid studied) is analogous. The common correlation relationships for all solid particles–liquids systems studied have been obtained. It has been found that the values of drag coefficient in both, the laminar and transitional ($Re_{0,n} < 165$) ranges of the flow for all fluids studied are determined by the generalised for power-law liquids Ergun equation. For the values of Reynolds number $Re_{0,n} > 165$ the friction factor values are lower than resulting from both, Ergun (1952) and Burke and Plummer (1928) relationships. It has been shown that the diagram proposed by Koziol et al. (1978) can be stated as the generalized one, not only for the determination of the solid particles motion in Newtonian fluids, but for the shear-thinning liquids too. In the last case it should be taken into account that the critical value of porosity cannot be taken equal to 0.4, but should be appropriate to the real porosity in the critical conditions for a given system solid particle–liquid (this one in the study presented was equal to 0.442). Furthermore, this experimental investigation permitted to generalize both, the map of Bi and Grace (1995) related to the characteristic fluidisation ranges and the diagram of the classification of particles fluidised proposed by Goossens (1998) for gas-fluidisation on any systems of solid particles–power law fluids. Therefore it has been found that the mathematical description of the fluidisation process is independent of the viscous fluid nature in which it is realised but depends on the correctly defined (taking into account the rheological properties) all of the characteristic dimensionless numbers and reduced diameters of the solid particles and minimal values of the velocities, characterizing the onset of the fluidisation.

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